International Journal of Material and Mechanical Engineering, 2012, 1: 8-15
Published Online April 2012

http://www.ijm-me.org

Transient Response of Planar Rigidly Jointed Truss Subjected to Step or Impulse Displacement Load

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(Abstract) Recently Pao *et al* proposed a method to solve the transient response of truss type structures under impact load: the method of reverberation ray matrix (MRRM). This is a novel matrix method in frequency domain that takes into account the multi-scattering of axial and bending waves at the joints and derives scattering coefficients upon equilibrium and compatibility conditions of the joints. The present work develops formulation of MRRM in terms of Euler-Bernoulli beam theory and analyzes the transient response of the structure subjected to step or impulse displacement load. The efficiency and accuracy of MRRM is verified by comparison of results of MRRM and Mindlin's solution for a single beam, and that of FEM for the transient response of axial and shear forces, bending moment, displacement, velocity and acceleration of a 17-bar rigidly jointed truss. In general, it is found that the result of MRRM still coincides with that of FEM. While, the solving process of MRRM is more simple and quick than that by FEM, and the MRRM needs less computing elements.

1. INTRODUCTION

In recent years, there has been an increasing interest on the dynamic response of truss or frame structures as need arises in outer space engineering, for instance, for the purpose of controlling local disturbance initiated by incident impact or monitoring of structure integrity of lattice type space station, as well as in civil engineering. There are mainly two approaches for the analysis of dynamic response of truss or frame. One is vibration analysis based on mode superposition approach, which finds the vibration modes of the structure and express the dynamic response by superposition of all components in the modal coordinates. The other one is to study the dynamic response of a structure by elastic wave theory. If the response is induced by impact load, then the initial stage of the transient response contains many high frequency components. The analyses based on wave propagation approach are shown to be superior to the other methods.

One of the early research work with ray tracing method was reported by Boley and Chao[1]. They studied how stress waves are generated by impact force at a joint of a truss, propagate along the members emanating from the loaded joint, and scatter from the joints at opposite ends. It was noted that by considering all possible paths of propagation the dynamic response of a truss can be described as a superposition of waves that reverberate within the truss. Only the first two or three reverberations of axial waves in their example were calculated due to lack of computer at that time. The scattering of incident axial wave at a rigid joint of three bars were analyzed and experimentally investigated by Desmond [2]. Experimental methodology for analyzing the effect of structural joints on wave propagation was presented by Doyle and Kamle[3]. A matrix formulation for planar frame was proposed by Nagem and Williams [4] which combines the transfer matrices with joint coupling matrices to form the system matrix that characterizes the steady state response of the structure. With the system matrix they have calculated the natural frequencies of a sample structure.

Recently, Howard and Pao[5]proposed a novel matrix method, i.e. the MRRM, which investigates the dynamic response of the planar trusses in terms of axial waves propagating along structural members and scattering at the joints. In this method the scattering coefficients representing the reflection and transmission of axial waves at each joint are derived from the equilibrium and compatibility conditions at the joint. Comparison between theoretical results and experiments indicate that the axial wave theory is only valid for the response at the very early time. In a following paper, based on Timoshenko beam theory the method is modified by Pao, Keh and Howard[6] to include the effect of flexural waves and mode

conversions between axial waves and flexural waves at the joint. Comparison to experimental data of the model truss under a step load shows good agreement for the early as well as considerably long time responses. Afterwards, Pao and Sun[7] applies MRRM to analyze the dynamic behavior of pin-jointed trusses subjected to suddenly applied force. It is found that the amplitude of dynamic bending strains in the members of pin-jointed trusses, as in the rigid jointed trusses, are comparable with the dynamic axial strains in the same member. Such a large dynamic bending strain in a truss member, which is contradictory to the static behavior of the truss, could only be caused by the inertial force of the mass in structural members of the truss.

In Ref. Pao *et al*[8] compared the method of transfer matrix (MTM) with MRRM in detail, concluded that MRRM is a viable alternative to the solutions of initial value and two point boundary problems of unidirectional wave motions and could be extended further to analyze wave propagation in two dimensional structures. Additionally, the MRRM is employed to obtain frequency response function (FRF) of displacement of a frame under the action of a unit impulse load by Miao *et al*[9]. The natural frequencies of the frame are determined from the FRF, the vibration mode is retrieved from the adjoint matrix of the coefficient matrix of the governing equations of MRRM. The MRRM has advantage over numerical methods, such as FEM.

The investigations of dynamic response of truss type structures with MRRM are still limited in the literature. Nothing else than the responses of bending and axial strains of trusses subjected to Heaviside step force have been reported. However, some of the engineering problems, such as sudden subsidence of foundation of engineering structures, can be categorized as suddenly applied displacement at support or joint of the structure. In the present study, the formulation of MRRM is re-derived based on Euler-Bernoulli beam theory that is a reduced case of the truss made of laminated beams. The latter is to be analyzed in our further research. The transient response of planar rigidly jointed truss subjected to step or impulse type displacement load is addressed with MRRM. The efficiency and accuracy of MRRM are demonstrated by comparing solutions of MRRM with results obtained by FEM.

2. THEORETICAL ANALYSIS

A planar truss-type structure is an assemblage of rods or beams pin-connected or rigidly connected at joints. In the following only rigidly jointed truss is considered. We define joints by letters (I, J, K, $\dots$) or numbers (1, 2, 3, $\dots$)and members by two letters or two numbers (IJ, JK, $\dots$; 12, 23, $\dots$). As shown in Fig.1, member JK means one with joint J as starting point and joint K as the end. The origin of a local coordinates $x^{JK}-y^{JK}$ locates at *J* with x^{JK} -axis laid along *JK*. The number of members meeting at the joint J is n^J. *X-Y* is the global coordinate system.

The governing differential equations based on Euler-Bernoulli beam theory for a uniform beam in frequency domain can be written as:

$$u(x,\omega)+\frac{E}{\rho\omega^2}\frac{\partial^2 u(x,\omega)}{\partial x^2}=0 \tag{1}$$

$$v(x,\omega)-\frac{EI}{\rho A\omega^2}\frac{\partial^4 v(x,\omega)}{\partial x^4}=0 \tag{2}$$

where, E, ρ, A, I are elastic modulus, density, cross sectional area and moment of inertia; u, v are axial and transverse displacements in the local *x*-*y* coordinate system. Then, axial force F, shear force Q bending moment M and angle of rotation φ of the member can be expressed as:

$$F=EA\frac{\partial u}{\partial x},\ \ Q=-EI\frac{\partial^3 v}{\partial x^3},\ \ M=EI\frac{\partial^2 v}{\partial x^2},\ \ \varphi=\frac{\partial v}{\partial x}; \tag{3}$$

The solutions for $u(x,\omega)$ and $v(x,\omega)$ are assumed as:

$$u=a_1e^{ik_1x}+d_1e^{-ik_1x} \tag{4}$$

$$v=a_2e^{ik_2x}+d_2e^{-ik_2x}+a_3e^{ik_3x}+d_3e^{-ik_3x} \tag{5}$$

$$\varphi=\frac{\partial v}{\partial x}=ik_2a_2e^{ik_2x}-ik_2d_2e^{-ik_2x}+ik_3a_3e^{ik_3x}-ik_3d_3e^{-ik_3x} \tag{6}$$

where $a_i(\omega)$, $d_i(\omega)$ ($i=1,2,3$) are the amplitude of arriving wave and departing wave and $k_i(\omega)$ (i =1,2,3) the wave number of the *i*-th mode, respectively. Substituting eqs.(4)~(6) into eqs.(1)~ (2), the wave number can be written as:

$$k_1=\sqrt{\frac{\rho}{E}}\omega,\quad k_2=\sqrt[4]{\frac{\rho A}{EI}}\sqrt{\omega},\quad k_3=i\cdot\sqrt[4]{\frac{\rho A}{EI}}\sqrt{\omega} \tag{7}$$

The external forces and moment applied at the joint J are denoted as f_X^J, f_Y^J, m^J. In the global coordinate system, the displacement in the *X* and the *Y* directions and the rotation of the joint J are denoted as $U^J(\omega)$, $V^J(\omega)$ and $\Phi^J(\omega)$, respectively. θ^{JK} is the orientation of the member *JK* with respect to X axis. The equations for the balance of forces along the *X* and the *Y* directions and the moments at the joint J are:

$$\sum_{K=1}^{n^J}[F^{JK}(0,\omega)\cos\theta^{JK}-Q^{JK}(0,\omega)\sin\theta^{JK}]+f_X^J=0 \tag{8}$$

$$\sum_{K=1}^{n^J}[F^{JK}(0,\omega)\sin\theta^{JK}+Q^{JK}(0,\omega)\cos\theta^{JK}]+f_Y^J=0 \tag{9}$$

$$\sum_{K=1}^{n^J}M^{JK}(0,\omega)+m^J=0 \tag{10}$$

The compatibility conditions of the displacements along the X and the Y directions and the rotation at the joint J are:

$$u^{JK}(0,\omega)\cos\theta^{JK} - v^{JK}(0,\omega)\sin\theta^{JK} = U^J(\omega) \quad (K=1,2,\ldots,n^J) \qquad (11)$$

$$u^{JK}(0,\omega)\sin\theta^{JK} + v^{JK}(0,\omega)\cos\theta^{JK} = V^J(\omega) \quad (K=1,2,\ldots,n^J) \qquad (12)$$

$$\varphi^{JK}(0,\omega) = \Phi^J(\omega) \quad (K=1,2,\ldots,n^J) \qquad (13)$$

The equilibrium equations and displacement compatibility conditions at joint J of the truss can be expressed in the matrix form as Ref.[7]:

$$\begin{bmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix}^{\mathbf{J}} \begin{Bmatrix} \mathbf{d}^{\mathbf{J}} \\ \mathbf{H}^{\mathbf{J}} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}^{\mathbf{J}} \left\{ \mathbf{a}^{\mathbf{J}} \right\} - \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}^{\mathbf{J}} \end{Bmatrix} \qquad (14)$$

For the above equations, vectors and matrixes are in bold font. They are all functions of frequency ω. {dJ} and {aJ} are vectors of amplitudes of departing and arriving waves for all members jointed at the joint J. The superscript J appearing in matrix [D] and [A] means they are the coefficient matrix of the joint J. Those coefficients inside square brackets consist of material parameters and geometric parameters of the truss. {HJ} and {fJ} consist of the unknowns and known quantities of the joint J in the global coordinate system respectively. For non-constrained degree of freedom of the joint, the displacement (angle of rotation) is unknown while the force (moment) is given. For constrained degree of freedom of the joint, the constraint force is unknown while the displacement of the joint is given. Solving eq.(14), the amplitudes of departing waves can be expressed by the amplitudes of arriving waves and the local source vector s^J at the joint J. It can be written in the matrix form as:

$$\mathbf{d}^{\mathbf{J}} = \mathbf{S}^{\mathbf{J}}\mathbf{a}^{\mathbf{J}} + \mathbf{s}^{\mathbf{J}} \qquad (15)$$

where $\mathbf{S}^{\mathbf{J}}$ is named as the local scattering matrix of the joint J. Assembling amplitudes of departing waves for all joints into a global vector $\mathbf{d}$, and amplitudes of the arriving waves into $\mathbf{a}$, the global equation relating **a** and **d** can be written as

$$\mathbf{d} = \mathbf{S}\mathbf{a} + \mathbf{s} \qquad (16)$$

The matrix $\mathbf{S}$ is referred to as the global scattering matrix, and the vector **s** as the global source vector. It is shown in Ref. [6] that the vector **a** is related to the vector **d** as

$$\mathbf{a} = \mathbf{PUd} \qquad (17)$$

in which **P** is the propagation matrix and **U** is the permutation matrix. Equation (16) then becomes

$$\mathbf{d} = \mathbf{SPUd} + \mathbf{s}$$

This equation can be reduced to

$$\mathbf{d} = [\mathbf{I} - \mathbf{R}]^{-1}\mathbf{s} \qquad (18)$$

where **R = SPU** is named as the reverberation ray matrix for the structure. It is noticed that the Fourier inverse transform of the matrix $[\mathbf{I}-\mathbf{R}]^{-1}$ involves an infinite number of poles along the real axis of the complex ω plane. To circumvent this difficulty, Pao *et al*[6] proposed that $[\mathbf{I}-\mathbf{R}]^{-1}$ is expanded into a Neumann series.

$$[\mathbf{I}-\mathbf{R}]^{-1} = \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \cdots + \mathbf{R}^n + \cdots \qquad (19)$$

Substituting Esq. (18) and (19) into Esq. (3) ~ (6), we can obtain the forces, the moment, the displacements and the rotation at any location of the member *JK* in the frequency domain. The unknowns of the joint *J* {$\mathbf{H}^{\mathbf{J}}$} can also be solved, since **d** and **a** have already known. Then, applying Fourier inverse transform, we can obtain the dynamic response of above variables in the time domain.

3. EXAMPLE OF A SIMPLY SUPPORT BEAM

To verify the theory and the computer code of MRRM, we first analyze a simple structure by the established theory. A simply supported beam is shown in Fig2. Assume the beam is made of aluminum alloy with square cross section. The side length of the cross section is 0.635cm. The elastic modulus of aluminum is 69GPa, density is 2700Kg/m^3, and axial wave speed is c_1=5039m/sec. Take l_o=0.1016m as a unit of length, the length of the beam is l =0.4064m (l=4l_0). A step displacement expressed using Heaviside function $\delta \cdot H(t)$ (δ=0.01l_0) is applied to the right end of the beam. The unit of time is taken as $t_0 = l_0/c_1 = 20.16\mu s$, which is the time needed for axial wave to propagate one unit of length. With MRRM, the beam is discretized into two equal-size elements. Joint 2 is at the mid-span of the beam.

Applying Mindlin's method[10], the transverse displacement of beam has the form[7]:

$$w(x,t) = [\frac{x}{l} + \sum_{n=1}^{\infty}(-1)^n \frac{2}{n\pi}\sin\frac{n\pi x}{l}\cos(\omega_n t)]\delta \qquad (20)$$

where $\omega_n = (n\pi/l)^2\sqrt{EI/\rho A}$. The results of transverse displacement at mid-span of the beam obtained with MRRM are compared with that of eq.(20) in Fig.3. In the figure the solid curve *v*23 is the result of displacement at the starting joint 2 of member 23 in the local coordinates, obtained from eq.(5) and evaluated at x = 0. The dashed curve *V*2 is the result of vertical displacement of Joint 2 in the global coordinate system, which is obtained as joint unknown in {$\mathbf{H}^{\mathbf{J}}$}. *V*2 should be equal to *v*23(0) by the compatibility condition eq.(12). The dotted curve is the result of eq.(20) with x =0.5l and 40 terms are included in series expansion. It is found that the first two curves resulted from the two solutions of MRRM are superposed and coincide very well with that of Mindlin's method.

4. TRANSIENT RESPONSE OF A 17-BAR FRAME SUBJECTED TO SEMI-SINE DISPLACEMENT IMPLUSE

In this section the laboratory model of the 17-bar aluminum rigidly-jointed truss, which is experimentally and analytically investigated in Ref.[7], is used as example. The material properties and cross sectional dimension are the same as the beam considered in the previous section. As shown in Fig.4, the length of the horizontal member (bar 35, 24, etc.), the vertical member (bar 34, 56 etc.) and the diagonal member (bar 45 etc.) are $3l_0$, $4l_0$ and $5l_0$, respectively. The total span of this frame is $L = 4\times 3l_0 = 1.219\mathrm{m}$. The unit of time is also t_0. As shown in Fig.5, the impact semi-sine displacement applied at joint 6 is $V_6 = \delta\cdot\sin(\pi t/\tau)$, in which $\delta = 0.01 l_0$, $\tau = 9t_0$. From 9 to 30 units of time, displacement amplitude is zero. In the frequency domain the semi-sine displacement impulse can be expressed as

$$V_6(\omega) = \frac{\delta\cdot\pi/\tau}{(\pi/\tau)^2-\omega^2}(1+e^{-i\omega\tau}) \qquad (21)$$

According to the above theory, we can calculate the transient displacement, force and moment at any location of each member, as well as the joint unknowns by inverse fast Fourier transform.

4.1. EQUILIBRIUM OF FORCES AND BENDING MOMENTS AT A JOINT

In the general theory the forces and the bending moments of all members connected at a joint are balanced at all times, as stated in eqs.(8)~(10). In Fig.6, the *X* component of the end force at the stating point of members 53, 54, 56 and 57 (i.e. at the point connecting to joint 5) are plotted. They are denoted as $F_X 53$, $F_X 54$, $F_X 56$ and $F_X 57$ respectively in Fig.6. Each of them is the sum of projections of axial force and shear force for the corresponding member when x = 0 in the local coordinates, as shown by the two terms in bracket of eq.(8). The *Y* component of the end force of these members are denoted as $F_Y 53$, $F_Y 54$, $F_Y 56$ and $F_Y 57$, and plotted in Fig.7. The unit of force is $f_0 = 712\mathrm{N}$. In Fig.8 the bending moments of the four members at joint 5 are plotted. The unit of bending moment is $m_0 = 72.36\mathrm{N\cdot m}$. The summations of these forces in *X* or in *Y* directions and the moments should be zero according to equilibrium condition of eqs.(8-10). It is found that the sums of the forces in either *X* or *Y* direction as well as the bending moments from the four members connected at joint 5 are balanced very well, which are depicted by the dashed straight line along the time axis.

4.2. COMPARISON WITH THE RESULTS OF FEM

In this section the results of MRRM are compared with that of FEM. In FEM, each member is discretized into 100 elements. While for MRRM the variables at any position of the member can be evaluated without discretization of any member. Figures 9~11 show the comparison of axial force, shear force and bending moment at the middle point of member 57 obtained from MRRM and from FEM. There are 2 curves in each figure. The curve in solid line is the result of MRRM, that in dotted line is the result of FEM, the unit of force and moment are also $f_0 = 712\mathrm{N}$ and $m_0 = 72.4\mathrm{N\cdot m}$. The results of MRRM and FEM show good agreement.

Figure 12 show the comparison of vertical displacement at joint 5 resulted from MRRM and FEM. In the figure *v*53, *v*54, *v*56 and *v*57 are the vertical component of displacement of the corresponding member connected at joint 5. Each of them is the sum of projections in *Y* direction of axial and transverse displacement *u* and *v* for the corresponding member when x = 0 in the local coordinates. They are found to be dispersed slightly. *V*5 is the vertical displacement of joint 5 calculated as joint unknown variable from $\{\mathbf{H}^\mathbf{J}\}$. *V*5_FEM is the curve of vertical displacement of joint 5 obtained from FEM. In general these results coincide very well. It is noticed that the maximum displacement of joint 5 along the *Y* direction is about $0.012 l_0$, which is larger than the amplitude of displacement inputted at joint 6 ($0.01\, l_0$).

Shown in Fig.13 are vertical velocities calculated from MRRM and FEM. One unit of velocity is $v_0 = 5039\mathrm{m/s}$. The first four curves denoted as *vel*53, *vel*54, *vel*56 and *vel*57 are corresponding responses in *Y* direction of the global coordinate system of the four members at joint 5, which are converted from solutions of axial and transverse velocities in the local coordinate system. The curve denoted as *Vel*5_FEM is the response of vertical velocity of joint 5 resulted from FEM. It is found that there is no response during the initial $4t_0$ time before substantial response is observed. $4t_0$ is the time for the disturbance to travel from joint 6 to joint 5.

Figure 14 compares results of vertical acceleration at joint 5 obtained from MRRM and FEM. One unit of acceleration is $a_0 = 2.49\times 10^8\,\mathrm{m/s^2}$. The first four curves denoted as *acc*53, *acc*54, *acc*56 and *acc*57 are corresponding accelerations in *Y* direction of the global coordinate system of the four members at joint 5, which are converted from solutions in local coordinate system. It is found that they are completely superposed. The curve denoted as *Acc*5_FEM is the result of FEM. In general, it still coincides with the results of MRRM. While, it also can be found that the solving process of MRRM is more simple and quick than that by FEM, and the MRRM needs less computing elements.

5. TRANSIENT RESPONSE OF THE 17-BAR TRUSS SUBJECTED TO STEP DISPLACEMENT

In figures 15 and 16 the vertical velocity and vertical acceleration at joint 5 are presented for the 17-bar rigidly jointed truss when it is subjected to a step vertical displacement $\delta\cdot H(t)$ (δ=0.01l_0) at joint 6. It is found that in these two figures the first four curves denoted as *vel*53, *vel*54, *vel*56,

*vel*57 in Fig.15 and *acc*53, *acc*54, *acc*56, *acc*57 in Fig.16 respectively, which come from the solution in the local coordinates of MRRM of each member connected at joint 5, are completely superposed. However, more disagreement between solutions of MRRM and FEM for the truss subjected to step displacement are observed than the truss subjected to semi-sine displacement load as presented in the previous section.

6. CONCLUSION

The MRRM, which was recently proposed by Pao *et al*[6], has been proved both theoretically and experimentally an efficient approach for transient dynamic analysis of truss type structures. This is a matrix analysis method in frequency domain that is successful in resolving singularity problem by expanding the singular matrix in the left side of eq.(19) into Neumann series before variables are inversely transformed by IFFT to time domain. In the present work the MRRM is substantiated in dynamic analysis of rigidly jointed truss where the load is given by a step or an impulse of displacement at a joint. The analysis and the computing of MRRM are verified first by the agreement between the results of MRRM and Mindlin's method of the simply supported beam subjected to end step displacement. Then the balance of the forces and moments at the ends of members connecting at a joint of the 17-bar truss are demonstrated. The analysis and computing are further verified by agreement between the results of MRRM and that of FEM for force, moment, displacement, velocity and acceleration of the 17-bar truss subjected to step or semi-sine displacement load. Compared with the FEM, the same results can be obtained by MRRM with less computing elements, and the solving processes are more simple and quick.

7. ACKNOWLEDGMENTS

This work is supported by Ningbo Natural Science Foundation of China (No.2011A610080) and the Impact and Safety of Coastal Engineering Initiative, a COE Program of Zhejiang Provincial Government at Ningbo University (Grant No.zj1020)

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Provincial Government at Ningbo University (Grant No.zj1020).

Figure captions

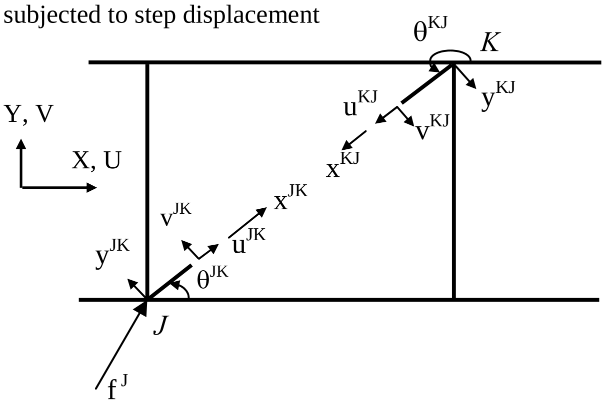

Fig.1 Local and global coordinate systems of the truss

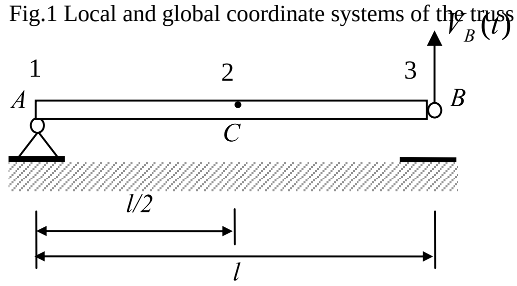

Fig.2 Simply supported beam subjected to step displacement at the right end

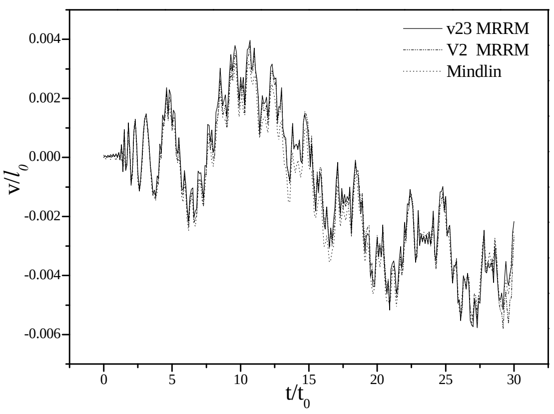

Fig.3 Comparison of vertical displacement at the middle point of the beam by MRRM and by Mindlin's method

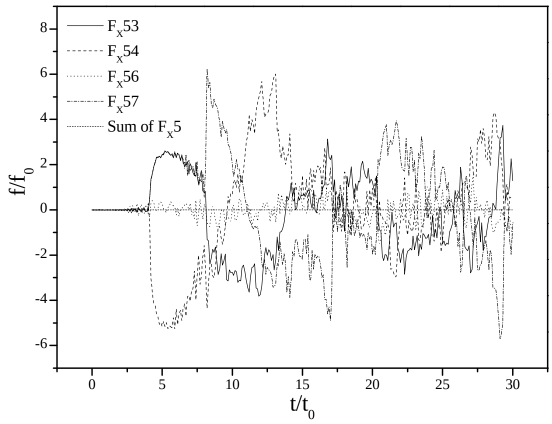

Fig.6 Equilibrium of forces in X direction at joint 5

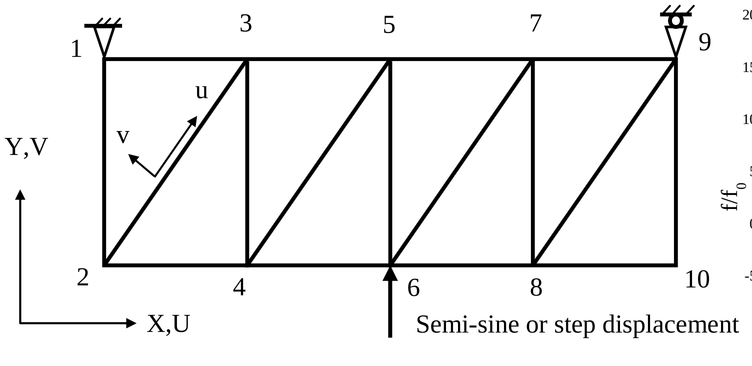

Fig.4 17-bar aluminum truss subjected to displacement load at joint 6

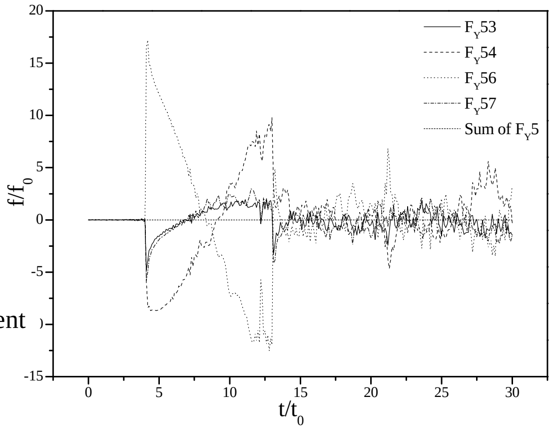

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Fig.7 Equilibrium of forces in Y direction at joint 5

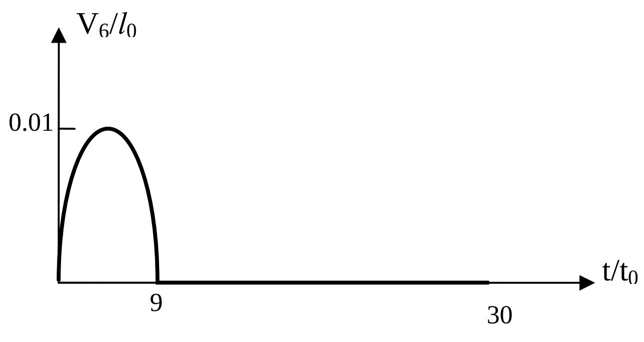

Fig.5 Load history of semi-sine displacement

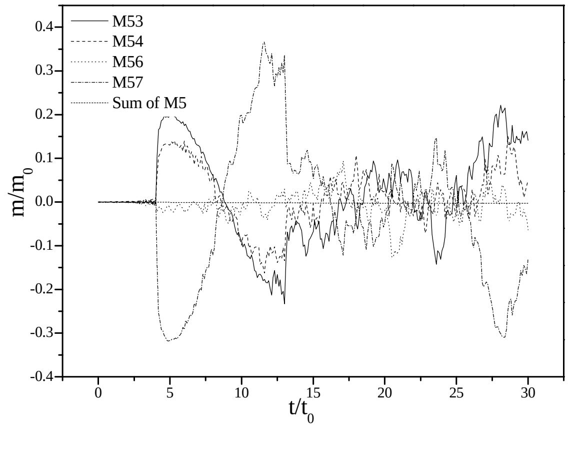

Fig.8 Equilibrium of bending moments at joint 5

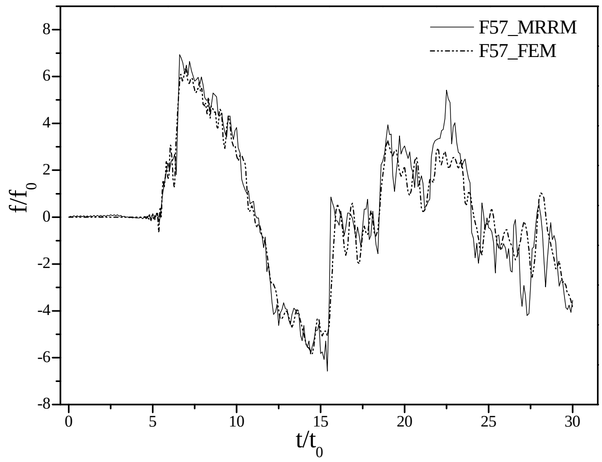

Fig.9 The dynamic response of axial force at the middle point of member57

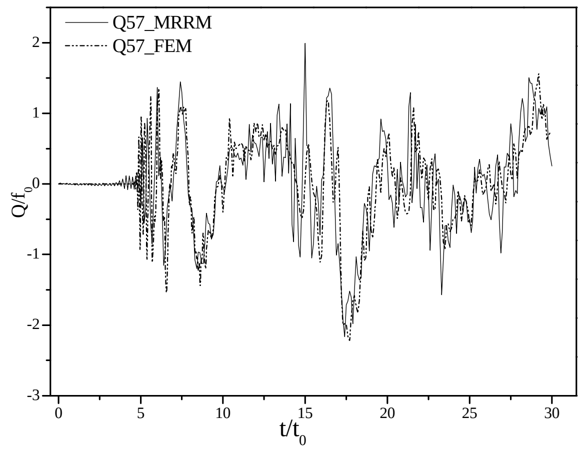

Fig.10 The dynamic response of shear force at the middle point of member57

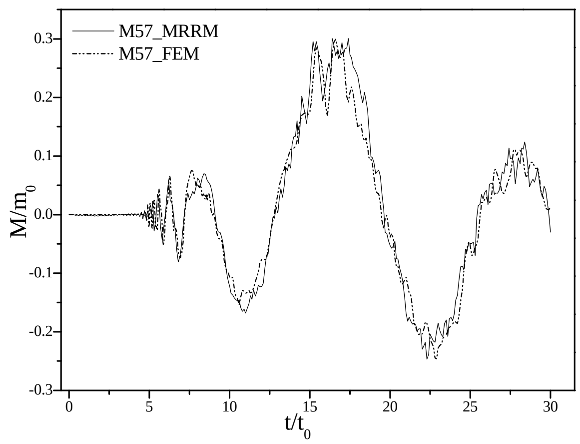

Fig.11 The dynamic response of bending moment at the middle point of member57

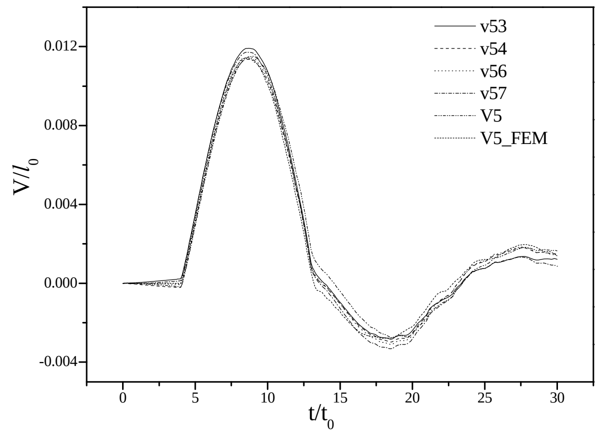

Fig.12 Vertical displacement at joint 5

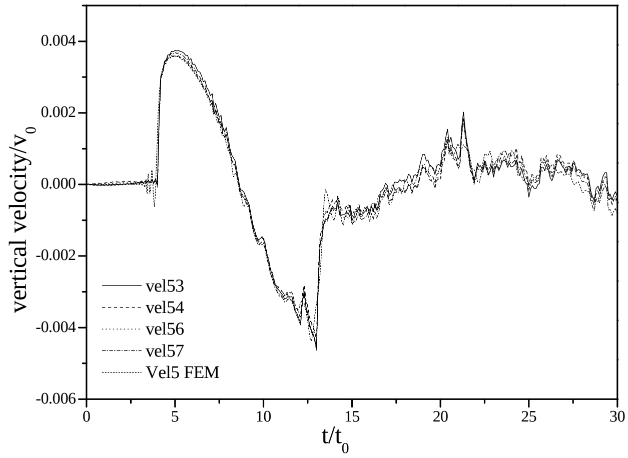

Fig.13 Vertical velocity at joint 5

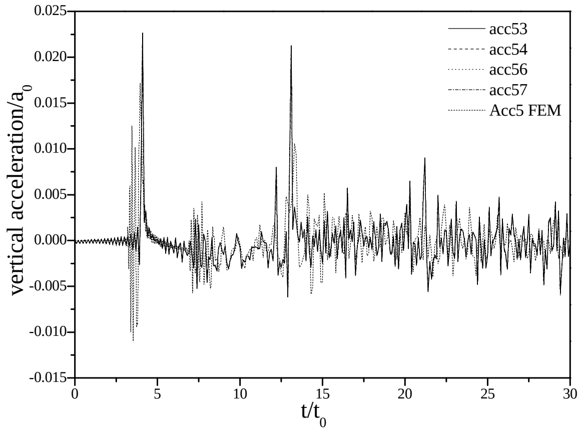

Fig.14 Vertical acceleration at joint 5

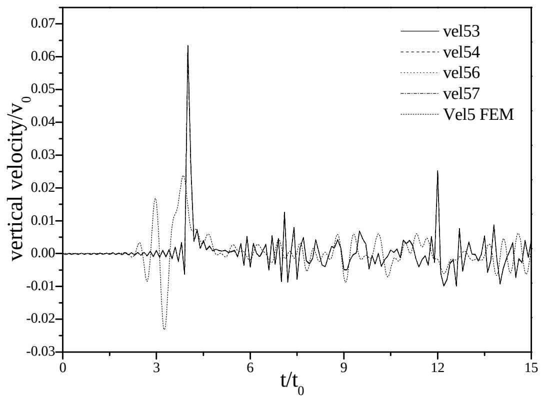

Fig.15 Vertical velocity at joint 5 when the truss subjected to step displacement

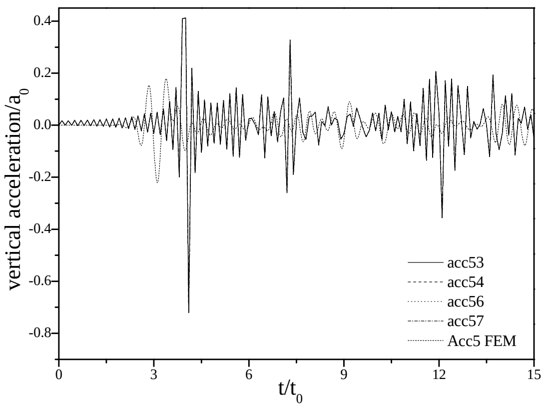

Fig.16 Vertical acceleration at joint 5 when the truss subjected to step displacement